(Pages : 4)

E – 1707

Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2018
First Degree Programme under CBCSS
MATHEMATICS
MM 1661.1 : Graph Theory (Elective)
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory.** They carry **1** mark **each**.

1. Define a loop.
2. Sum of degrees of vertices = _____
3. What are edge disjoint subgraphs ?
4. Define outdegree.
5. Is the following graph connected ?



6. Define Euler graph.
7. What is unicursal line ?
8. What is a tree ?
9. A tree with n vertices has _____ edges.
10. Define centre of a graph.

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SECTION – II

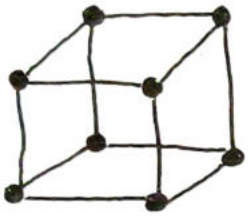
Answer **any 8** questions from among the questions **11** to **22**. These questions carry 2 marks **each**.

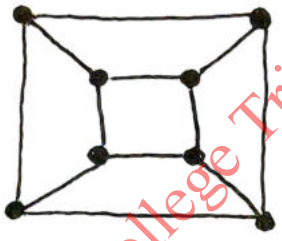
11. Write any two applications of graph theory with suitable explanation.

12. Write the adjacency matrix of K_4.

13. Prove that number of odd vertices in a graph is even.

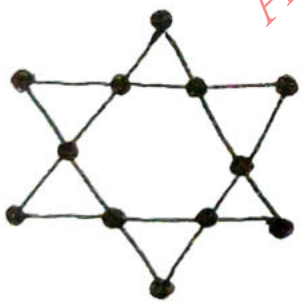
14. Label the following graphs to prove that they are isomorphic.





15. Prove that a graph G is disconnected if and only if the vertex set can be partitioned into 2 non-empty disjoint subsets V_1 and V_2 such that there is no edge having one end vertex in V_1 and another in V_2.

16. Is the following graph is Euler graph ? Explain



17. Explain Konigsberg bridge problem.

18. State a characterization theorem for Euler digraph. Illustrate with an example.

19. Prove that there is one and only one path between every pair of vertices in a tree T.

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20. Prove that a graph with n vertices, n – 1 edges and no circuits is connected.

21. Prove that in any tree T there are at least two vertices of degree one.

22. Define spanning tree. Find a spanning tree of the following graph.



SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Draw all non-isomorphic graphs on 4 vertices. How many of them are self -complementary ? How many are connected ?

24. Define spanning subgraph and induced subgraph. Is K_3 a spanning subgraph of K_4 ? Is it an induced subgraph ? Explain.

25. Define incidence matrix. Draw the graph with incidence matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

26. Prove that a graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ components.

27. Prove that G is an Euler graph if and only if every vertex is of even degree.

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28. Prove that every tree has either one or two centers.

29. Prove that every connected graph has at least one spanning tree.

30. Draw planar representations of K_5 minus an edge and a cube.

31. Prove that in any simple connected planar graph with f regions, n vertices and e edges, $e \geq \frac{3f}{2}$ and $e \leq 3n - 6$.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. Explain in detail multicolour cube puzzle and its graph theoretic model.

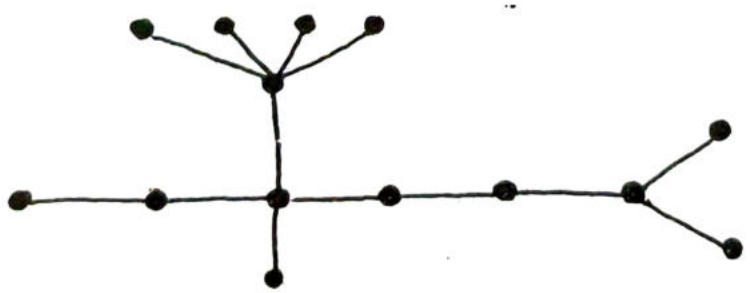
33. a) In a connected graph G with exactly 2k odd vertices, prove that there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

    b) Prove that a simple connected graph has at most $\frac{(n-1)(n-2)}{2}$ edges.

34. a) Prove that, if in a graph G there is one and only one path between every pair of vertices, then G is a tree.

    b) Prove that a tree with n vertices had n – 1 edges.

    c) Find the center of the following tree.



35. What are Kuratowski's graphs ? Prove that they are non-planar.

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